SYSTEM AND METHOD FOR HETERODYNE INTERFEROMETER HIGH VELOCITY TYPE NON-LINEARITY COMPENSATION

Cross Reference to Related Applications

This application is related to U.S. Patent Application Serial Number 10/003,798, filed October 26, 2001, entitled PHASE DIGITIZER, and U.S. Patent Application Serial Number 10/010,175, filed November 13, 2001, entitled SYSTEM AND METHOD FOR INTERFEROMETER NON-LINEARITY COMPENSATION.

The Field of the Invention

This invention relates generally to interferometer systems. This invention relates more particularly to a system and method for heterodyne interferometer compensation of a type of non-linearity occurring at high velocity.

Background of the Invention

Light leakage between beams in an interferometer in metrology produces measurement results that depart periodically from the ideal – known as non-linearity.

One prior art technique for compensating for non-linearity in a homodyne interferometer involves balancing the two in-phase and quadrature signals within the interferometer for offset, gain and orthogonality. The signals are digitized, computed for imbalance, and analog electronics are used to inject offset and gain compensations. The compensation components are all contained within the interferometer itself. In a heterodyne interferometer, there are no in-phase and quadrature signals to balance. Thus, such a compensation method is not applicable to a heterodyne interferometer.

It would be desirable to provide a system and method for non-linearity compensation of interferometer position data using digital numerical processing, wherein the non-linearity is of a type that occurs at high velocities.

Summary of the Invention

One form of the present invention provides a method for compensating non-linearity of the high-velocity type manifested in heterodyne interferometer position data. A plurality of groups of digital position values are received. A plurality of groups of digital phase values from a measurement channel are received. A first group of the digital position values and digital phase values are digitally processed to generate a plurality of block data values. The plurality of block data values are digitally processed to generate at least one quasi-static non-linearity parameter. A second group of the digital position values are compensated based on the at least one quasi-static non-linearity parameter.

Brief Description of the Drawings

Figure 1 is an electrical block diagram illustrating a prior art displacement measuring heterodyne interferometer system.

Figure 2 is a functional block diagram illustrating a high velocity non-linearity compensation system according to one embodiment of the present invention.

Figure 3 is a diagram illustrating a position-data word output by the interferometer system of Figure 1.

Figure 4 is an electrical schematic diagram illustrating one embodiment of the block-data generator shown in Figure 2.

Figure 5A is an electrical schematic diagram illustrating a modulo-320 counter for generating counter signals for controlling the operation of the block-data generator shown in Figure 4.

Figure 5B is a diagram illustrating control signals for controlling the operation of the block-data generator shown in Figure 4.

Figure 6 is a diagram of a group of 320 position-data words partitioned into 10 blocks of 32 words each.

Figure 7 is an electrical block diagram illustrating one embodiment of the compensation processing block shown in Figure 2.

Description of the Preferred Embodiments

In the following detailed description of the preferred embodiments, reference is made to the accompanying drawings, which form a part hereof, and in which is shown by way of illustration specific embodiments in which the invention may be practiced. It is to be understood that other embodiments may be utilized and structural or logical changes may be made without departing from the scope of the present invention. The following detailed description, therefore, is not to be taken in a limiting sense, and the scope of the present invention is defined by the appended claims.

I. DISPLACEMENT MEASURING INTERFEROMETRY SYSTEM

The non-linearity compensation system and method of the present invention is discussed in the context of a heterodyne displacement measuring interferometry system.

A typical displacement measuring interferometer system consists of a frequency-stabilized laser light source, interferometer optics and measuring electronics. In metrology based on homodyne interferometry, the phase progression function $\phi(t)$ is directly proportional to the object displacement in time, t, usually by the factor $\lambda/4$. That is, one unit interval (UI) change represents an object movement of one-quarter of the wavelength of the light wave. One UI represents one light wave interference fringe, or 2π radians. In mixing, phase is preserved, a travel of $\lambda/4$ is manifested as one interference fringe. In metrology based on heterodyne interferometry, there are two channels: one Doppler-shifted (Measurement Channel), and the other not shifted (Reference Channel). The difference between the two phase progression functions $\phi_M(t)$ and $\phi_R(t)$ of the two channels is proportional to the object displacement to within an arbitrary constant. The phase-progression functions for both channels are monotonically increasing with time.

Figure 1 is an electrical block diagram illustrating a prior art heterodyne displacement measuring interferometer system 100. Interferometer system 100 includes laser 102, interferometer 108, measurement and processing electronics 112, and fiber optic pickup 114. Interferometer 108 includes stationary retroreflector 104, polarizing beam splitter (PBS) 106, and movable retroreflector 110.

Laser 102 generates a pair of collinear, orthogonally-polarized optical beams of equal intensity and of different frequencies F_1 and F_2, which differ in frequency by F_R, which is a reference frequency (also referred to as a split frequency). The optical beams pass through interferometer 108. Polarization beam splitter 106 reflects one polarization of the incoming light to stationary retroreflector 104, and passes the other polarization of light to movable retroreflector 110. The retroreflectors 104 and 110 return the light to the polarization beam splitter 106, where one beam is transmitted and the other beam is reflected, so that the two beams are again collinear. Linear motion of the movable retroreflector 110 results in a corresponding change in the difference in phase between the two beams. The output beams from interferometer 108 go to fiber optic pick-up 114. In fiber optic pick-up 114, the output beams from interferometer 108 are mixed, and the mixed beam is coupled to an optical fiber 113. The mixed beam is referred to as the measurement signal, and the mixing is represented by the following Equation I:

Equation I

Measurement signal = $\overleftrightarrow{F}_1 \otimes F_2$

where:

$\otimes$ indicates a mixing operation; and

the overhead line on F_1 indicates that the signal is Doppler-shifted.

Measurement and processing electronics 112 contain a fiber optic receiver that produces an electrical measurement signal corresponding to the optical measurement signal. The measurement signal has a frequency that is

equal to the reference frequency, F_R, plus the Doppler shift frequency, as shown in the following Equation II:

Equation II

$$F_M = F_R + nv/\lambda$$

where:

v is the velocity of the interferometer element whose position is being measured (the sign of v indicates the direction of travel);

λ is the wavelength of light emitted from laser 102; and

n equals 2, 4, etc., depending on the number of passes the light makes through interferometer 108. In embodiments of the present invention, n=4.

In the example system of Figure 1, the movement of retroreflector 110 produces the Doppler shift and n is equal to 2. Laser 102 also outputs a reference signal at the reference frequency (F_R) via a fiber optic cable 111 that goes to a fiber optic receiver in the measurement and processing electronics 112. The reference signal is produced by mixing the two beams from laser 102 (F_1 and F_2), which is represented by the following Equation III:

Equation III

$$\text{Reference Signal} = F_1 \otimes F_2$$

Measurement and processing electronics 112 contain a fiber optic receiver that produces an electrical reference signal corresponding to the optical reference signal. The reference signal has a frequency that is equal to the reference frequency F_R.

Measurement and processing electronics 112 measure and accumulate the phase difference between the reference signal and the measurement signal, and process the difference to provide position and velocity outputs.

II. NON-LINEARITY

In less than ideal situations, light leakage between beams from laser 102 occurs, causing a small amount of one frequency to be present in the other

frequency. Symbolizing the ideal by a capital letter and leakage by a lower-case letter, the non-ideal situation can be symbolized by the following Equation IV and Equation V:

<u>Equation IV</u>

$$\text{Measurement Signal}: (\overleftrightarrow{F_1} + \overleftrightarrow{f_2}) \otimes (F_2 + f_1)$$

<u>Equation V</u>

$$\text{Reference Signal}: (F_1 + f_2) \otimes (F_2 + f_1)$$

Through mixing, signals of many frequencies are produced. The measurement signal has six mixed components:

$$\text{M1}: \overleftrightarrow{F_1} \otimes F_2;\ \text{M2}: \overleftrightarrow{F_1} \otimes \overleftrightarrow{f_2};\ \text{M3}: f_1 \otimes F_2;\ \text{M4}: f_1 \otimes \overleftrightarrow{F_1};\ \text{M5}: \overleftrightarrow{f_2} \otimes F_2;\ \text{M6}: f_1 \otimes \overleftrightarrow{f_2}$$

The reference signal also has six mixed components:

$$\text{R1}: F_1 \otimes F_2;\quad \text{R2}: F_1 \otimes f_2;\quad \text{R3}: f_1 \otimes F_2;\quad \text{R4}: f_1 \otimes F_1;\quad \text{R5}: f_2 \otimes F_2;\quad \text{R6}: f_1 \otimes f_2$$

The signals *M1* and *R1* are the desired measurement and reference signals, respectively, not non-linearity. Typically, *M1* and *R1* are the dominant components.

The signals *M6* and *R6* are each mixing products of two typically small signals, producing only second-order effects. Signals *M6* and *R6* may be ignored.

The signals *R2, R3, R4*, and *R5* are static parameters. The signals *R2* and *R3* have the same frequency as the ideal signal *R1*, which is the reference or split frequency. The combined effect of *R2* and *R3* is to cause an inconsequential constant phase shift to the reference signal. Signals *R4* and *R5* are "steady" signals mixing with themselves, resulting only in a static DC amplitude shift. A "steady" signal is a signal that is not Doppler shifted. In an AC-coupled circuit, *R4* and *R5* produce no effect, and may be ignored.

Non-linearity caused by *M2* and *M3* is most obvious at moderate velocities (*e.g.*, under about 50 mm/sec). *M2* and *M3* produce an interference signal at the Reference frequency that perturbs the measurement signal, *M1*, to cause non-linearity. The temporal frequency of the non-linearity is the Doppler frequency, and therefore the spatial frequency of the non-linearity is invariant (it is always one period in $\lambda/4$ of the distance traveled). Non-linearity caused by *M2* and *M3* is periodic with the position data with a period of 1 UI, or 1 "fringe," representing $\lambda/4$ of the distance traveled. For a HeNe laser, λ is 633 nanometers. The spatial frequency invariance property of *M2* and *M3* type of non-linearity facilitates compensation.

The signals *M4* and *M5* are a type non-linearity that is observed at high speed of the measurement object away from the light source. The mixing product of *M4* has the form shown in the following Equation VI:

<u>Equation VI</u>

$$\cos 2\pi(F_1 t - \frac{p_1}{\lambda/4}) \otimes \cos 2\pi(F_1 t) \Rightarrow \cos 2\pi(\frac{p_1}{\lambda/4})$$

where:

p_1 is a position parameter; and

λ is the wavelength of light emitted from laser 102

Similarly, the mixing product of *M5* has the form shown in the following Equation VII:

<u>Equation VII</u>

$$\cos 2\pi(F_2 t - \frac{p_2}{\lambda/4}) \otimes \cos 2\pi(F_2 t) \Rightarrow \cos 2\pi(\frac{p_2}{\lambda/4})$$

where:

p_2 is a position parameter; and

λ is the wavelength of light emitted from laser 102

The position-parameters p_1 and p_2 may have different origins, but are almost identically scaled due to F_1 being only a few megahertz from F_2 in one embodiment. The temporal frequency of *M4* and *M5* is the same, and equals the

Doppler frequency, $f_{Doppler} = f_R - f_M$. The *M4* and *M5* signals perturb the measurement signal, at $f_M - f_{Doppler}$, producing non-linearity at the temporal frequency of $2^*f_M - f_R$. In one embodiment, *M4* and *M5* type of non-linearity can be seen only when the measurement frequency nears half the reference frequency, which occurs when the object is moving rapidly (*e.g.*, typically about 0.5 m/sec) within a small range of velocities away from the source. Unfortunately, neither the temporal nor spatial frequency of *M4* and *M5* type of non-linearity is invariant, making its compensation more complex. In addition to the position-data, the measurement channel frequency is needed. One embodiment of the present invention compensates non-linearity caused by the signals *M4* and *M5*.

The non-linearity perturbation, $\Delta\phi(t)$, (in UI), as a function of time, is shown in the following Equation VIII:

Equation VIII

$$\Delta\phi(t) = \left(\frac{1}{2\pi}\right)\arctan\left[\frac{r\cos 2\pi[(2f_M - f_R)t - \theta]}{1 - r\sin 2\pi[(2f_M - f_R)t - \theta]}\right]$$

where:

r is the ratio of the magnitudes of the combined perturbing *M4*/*M5* signal to that of the ideal signal *M1*; and

θ is a phase-offset between the phase progression of $(2f_M - f_R)$ and the non-linearity pattern of the position data at the same frequency.

III. NON-LINEARITY COMPENSATION

A. Overview of Compensation System

In one embodiment of the present invention, the measured position data output by an interferometer system 100 and the phase progression of the measurement channel are processed, and two best-fit, quasi-static, non-linearity parameters are produced to compensate data in the near future. Continuing the process indefinitely, all data after an initial latency period are compensated. In

one form of the invention, non-linearity magnitude and phase parameters are determined on-the-fly from 320 consecutive position-data words and 320 measurement channel phase progression words, and are used to compensate the position-data words immediately following. One form of the invention is a completely digital process that continues in "leapfrog" fashion, compensating all future data using the periodically updated non-linearity parameters.

Figure 2 is a functional block diagram illustrating a non-linearity compensation system 200 according to one embodiment of the present invention. Figure 2 also shows a phase digitizing portion 100P of interferometer system 100. Phase digitizing portion 100P includes a measurement channel phase digitizer 208M, a reference channel phase digitizer 208R, and arithmetic logic unit (ALU) 207. Measurement channel phase digitizer 208M produces measurement channel phase data 209M based on a received measurement signal. Reference channel phase digitizer 208R produces reference channel phase data 209R based on a received reference signal. ALU 207 produces observed position data 204 based on the difference between measurement channel phase data 209M and reference channel phase data 209R.

Non-linearity compensation system 200 includes compensation processing block 206, block data generator 210, parameter generator 212, and ALUs 217 and 220. Compensation processing block 206 includes block RAM table 206A and ALU 206B. In one embodiment, non-linearity compensation system 200 compensates *M4* and *M5* type of non-linearity in uncompensated position data 202, which includes observed position data 204 output by interferometer system 100, and may also include expanded position data (e.g., expanded through extrapolation).

Observed position data 204 ($p(j)$) from interferometer system 100 are provided to ALU 220 and block data generator 210. ALU 220 also receives measurement channel phase data 209M from interferometer system 100. ALU 220 sums observed position-data 204 ($p(j)$) and measurement channel phase data 209M. The sum of observed position-data 204 ($p(j)$) and measurement channel

phase data 209M is the non-linearity phase progression 219 ($\psi_{NL}(j)$), which is provided to ALU 217 and block data generator 210.

In one embodiment, both observed position-data 204 ($p(j)$) and non-linearity phase progression 219 ($\psi_{NL}(j)$) are output at a 3.2 microsecond rate and are used by block-data generator 210 in groups of 320 to produce 21 sums for each group. These sums, in turn, are used by parameter generator 212 to construct non-linearity parameters 214M and 214P (collectively referred to as non-linearity parameters 214). The "j" in $p(j)$ is an index for identifying observed position-data words and corresponding words of measurement channel phase data 209M at a 3.2 microsecond rate. In one embodiment, non-linearity parameters 214 include a non-linearity (NL) magnitude parameter 214M (V_{NL}), and a NL phase offset parameter 214P (θ_{NL}). Block RAM table 206A is synthesized from the non-linearity magnitude parameter 214M (V_{NL}). The non-linearity phase offset parameter 214P (θ_{NL}) is provided to ALU 217.

ALU 217 subtracts the non-linearity phase offset parameter 214P (θ_{NL}) from the non-linearity phase progression 219 ($\psi_{NL}(j)$) to form offset-removed phase progression parameter 218 ($\phi_{NL}(j)$), which continuously addresses block RAM table 206A.

Uncompensated position data 202 is provided to compensation processing block 206. ALU 206B subtracts compensation values obtained from block RAM table 216A from uncompensated position data 202 to produce compensated position data. Incoming data 202 is non-linearity compensated at a higher, lower, or the same interferometer output rate of 3.2 microseconds. Meanwhile, "future" uncompensated, non-linearity phase progression data 219 ($\psi_{NL}(j)$) and position data 204 ($p(j)$) are used by block-data generator 210 and parameter generator 212 to generate yet further non-linearity parameters 214 to compensate for yet more future position data. After an initial latency period, all data will be compensated.

Since the non-linearity parameters 214M and 214P are quasi-static and continually updated, the process acts as a tracking filter, producing appropriate

parameters 214 from recent position data 204. In one form of the invention, the compensation performed by compensation processing block 206 is at nanosecond speed.

In one embodiment, block data generator 210 detects when unfavorable conditions for accurate measurement occur, and generates an update inhibit signal 216. When an update inhibit signal is generated, parameter generator 212 stops updating non-linearity parameters 214. During this time, position data is compensated by compensation processing block 206 using the existing non-linearity parameters 214. Updating of the non-linearity parameters 214 continues when measurement conditions improve.

In one embodiment, the functions performed by non-linearity compensation system 200 are limited to digital numerical computation by hardware, firmware, or a combination, and system 200 includes no optical or analog electrical circuitry. In one form of the invention, compensation system 200 is implemented using field programmable gate arrays (FPGAs) and/or one or more DSP processors.

Figure 3 is a diagram illustrating an observed position-data word 300 output by interferometer system 100. Position-data word 300 includes 32 bits. The most significant 22 bits (*W1-22*) of word 300 represent travel of multiples of one whole fringe of $\lambda/4$. The least significant 10 bits (*F1-10)* of word 300 represent the fractional portion of a fringe.

B. Block Data Generation

Figure 4 is an electrical schematic diagram illustrating one embodiment of the block-data generator 210 shown in Figure 2. Block data generator 210 includes cosine lookup table 402, sine lookup table 404, arithmetic logic units (ALUs) 406A-406U (collectively referred to as ALUs 406), bit shifter 408, multiplexer 410, bit shifter 412, multiplexer 414, bit shifter 418, multiplexer 420, registers 421, up counter 422, bit shifter 424, accumulator 426, and modulo-320 counter and signal generator 428. In one embodiment, the digital circuits shown in Figure 4 are clocked synchronously at a 3.2 microsecond rate.

The clocking circuit is omitted from Figure 4 to simplify the illustration of the invention.

Observed position-data words 300, which are 32 bits wide, arrive at 3.2 microsecond intervals at "word in" terminal 416. The fractional part of position-data words 300 is added to the fractional part of measurement channel phase progression data 209M ($\phi_M(j)$) at ALU 220. In one embodiment, block data generator 210 processes the received words of position-data 204 ($p(j)$) and measurement channel phase data 209M ($\phi_M(j)$) in groups of 320 words, and generates 21 output data blocks for each set of 320 words of position-data 204, and 320 words of measurement channel phase data 209M.

Modulo-320 counter and signal generator 428 includes modulo-320 counter 500 (shown in Figure 5A), and logic gates 510, 512, 514, 516 and 517 (shown in Figure 5B). The processing performed by block-data generator 210 is scheduled by modulo-320 counter 500. Modulo-320 counter 500 comprises a cascade of three counters -- modulo-32 counter 502, modulo-2 counter 504, and modulo-5 counter 506. Modulo-32 counter 502 and modulo-2 counter 504 are traditional binary up-counters. Modulo-5 counter 506 counts in a perpetually recurring sequence 3-4-5-6-7.

The clock-in input (Cin) of modulo-32 counter 502 is coupled to a 3.2 microsecond clock signal. The terminal count (TC) output of modulo-32 counter 502 is coupled to the clock-in input of modulo-2 counter 504. The output (Q) of modulo-32 counter 502 is the 5 least significant bits (Q0-Q4) of the counter 500. The terminal count output of modulo-2 counter 504 is coupled to the clock-in input of modulo-5 counter 506. The output (Q) of modulo-2 counter 504 is the fourth most significant bit (Q5) of counter 500. The output (Q) of modulo-5 counter 506 is the three most significant bits (Q6-Q8) of counter 500.

A group clock signal is produced every 1024 microseconds at the terminal count of modulo-5 counter 506. The four most significant bits (Q5-Q8) of counter 500, and the most significant two bits (*NLF1* and *NLF2*) of the fractional parts of the non-linear phase 219 (Ψ_{NL}) are logically combined to

generate 10 signals for controlling the 21 ALUs 406 as shown in the following Table I:

Table I

Signal #	Name	Logical Combination
1.	CE.D	$Q6 + (Q5 \oplus Q7)$
2.	SGN.D	1
3.	CE.L	Q6•Q7
4.	SGN.L	Q8
5.	CE.Q	Q6
6.	SGN.Q	Q7
7.	CE.24	*NLF2*
8.	CE.13	NOT(*NLF2*)
9.	SGN.24	NOT(*NLF1*)
10.	SGN.13	NOT(*NLF1*)

In the above Table I, "+" indicates a logical OR operation, "⊕" indicates a logical EXLUSIVE OR operation, and "•" indicates a logical AND operation.

Figure 5B is a diagram illustrating the 10 control signals listed in Table I, and the logic gates for generating some of the signals. As shown in Figure 5B, signal CE.D is generated by performing an EXCLUSIVE-OR operation on counter bits Q5 and Q7 with EXCLUSIVE-OR gate 510, and then performing a logical OR operation with OR gate 512 on counter bit Q6 and the output of EXCLUSIVE-OR gate 510. Signal SGN.D is a constant logical 1. Signal CE.L is generated by performing a logical AND operation on counter bits Q6 and Q7 with AND gate 514. Signal SGN.L is generated from counter bit Q8. Signal CE.Q is generated from counter bit Q6. Signal SGN.Q is generated from counter bit Q7. Signal CE.24 is generated from the second most significant fractional bit, *NLF2*, of non-linear phase 219 (Ψ_{NL}). Signal CE.13 is generated by inverting with inverter 516 the second most significant fractional bit, *NLF2*, of non-linear phase 219 (Ψ_{NL}). Signals SGN.24 and SGN.13 are generated from

the complement of the most significant fractional bit, *NLF1*, of non-linear phase 219 (Ψ_{NL}) through inverter 517 creating the complement.

Of the 10 signals shown in Figure 5B, the SGN.x signals are coupled to corresponding SGN.x (polarity) inputs of ALUs 406, and the CE.x signals are coupled to corresponding CE.x (clock enable) inputs of ALUs 406, where "x" identifies the particular SGN and CE signals (*e.g.*, D, L, Q, 13, or 24) that are coupled to a particular ALU 406. For example, as shown in Figure 4, ALU 406A includes clock enable input CE.D, and polarity input SGN.D, indicating that signals CE.D and SGN.D, respectively, are coupled to these inputs of ALU 406A. The connection lines are not shown in Figure 4 to simplify the illustration of the invention.

The counter arrangement 500 illustrated in Figure 5A effectively partitions a group of 320 input words 300 into ten sequential blocks of 32 words each, numbering block 1 through block 10. Figure 6 is a diagram of a group 600 of 320 position-data words 300, partitioned into 10 blocks 602A-602J (collectively referred to as blocks 602) of 32 words each.

A CE.x signal, when TRUE, enables corresponding ALUs 406. A SGN.x signal, when TRUE, dictates increment for an enabled ALU 406, and decrement when the signal is FALSE. The CE.x and SGN.x signals affect operation of ALUs 406 as shown in the following Table II:

<u>Table II</u>

<u>ALU's</u>	<u>Operation</u>
1. ALUs 406A (CD), 406F (SD), and 406K (PD)	D: increment in blocks 1, 2, 4, 5, 6, 7, 9, 10; disabled in blocks 3 and 8.
2. ALUs 406B (CL), 406G (SL), and 406L (PL)	L: decrement in blocks 1, 2; disabled in blocks 3, 4, 5, 6, 7, 8; increment in blocks 9,10.
3. ALUs 406C (CQ), 406H (SQ), 406M (PQ)	Q: increment in blocks 1, 2, 9, 10; disabled in blocks 3,

4, 7, 8; decrement by twice the values in blocks 5, 6.

As shown in the following Table III, regardless of the particular block 602 being processed, the following ALUs 406 operate only on the logical values of the most significant two fractional bits (*NLF1*, *NLF2*) non-linear phase 219 (Ψ_{NL}):

Table III

ALU's	Operation
1. ALUs 406D (C24), 406I (S24), 406N (P24), 406P (I24), 406R (J24), 406T (K24)	"24": increment on (0,1); decrement on (1,1); disabled on (0,0) and (1,0).
2. ALUs 406E (C13), 406J (S13), 406O (P13), 406Q (I13), 406S (J13), 406U (K13)	"13": increment on (0,0); decrement on (1,0); disabled on (0,1) and (1,1).

Words 300 of position-data 204 received on terminal 416, each 32-bits in width, are connected to the inputs of ALUs 406K (PD), 406L (PL), 406N (P24), and 406O (P13), bit shifter 418, and multiplexer 420. A second input (input B) of multiplexer 420 is the position-data word 300 multiplied by 2 through shifting by 1 bit with bit shifter 418. The multiplexed position-data output by multiplexer 420 is connected to ALU 406M (PQ). Multiplexer 420 is steered by the same control bit, SGN.Q, as ALU 406M (PQ). This arrangement results in the value of ALU 406M (PQ) being incremented by the position-data word 300 on TRUE SGN.Q, but decremented by twice the position data-word 300 on FALSE SGN.Q.

For summation purposes, the width of position-data words 300 can be reduced from 32 bits to 24 bits without penalty by subtracting a constant integer, such as the "whole" part of the last data word 300 from the last group 600, from each of the 320 subsequent position-data words 300.

The most significant 8 bits (*F1-F8*) of the fractional part of words of position-data 204 received on terminal 416 are added by ALU 220 to the fractional part of the measurement channel phase progression words 209M ($\phi_M(j)$). The sum, signified as $\psi_{NL}(j)$ (and referred to as non-linearity phase progression 219), addresses two look-up tables – cosine lookup table 402 and sine lookup table 404. Lookup tables 402 and 404 each span one complete period in the 8-bit address space. There are, therefore, 256 entries in each table 402 and 404 of one-period. Each entry in tables 402 and 404 is 10 bits wide.

The output of cosine table 402 is connected to the inputs of ALUs 406A (CD), 406B (CL), 406D (C24), and 406E (C13), bit shifter 408, and multiplexer 410. A second input (input B) of multiplexer 410 is the output of cosine table 402 multiplied by 2 through shifting by 1 bit with bit shifter 408. The multiplexed position-data output by multiplexer 410 is connected to ALU 406C (CQ). Multiplexer 410 is steered by the same control bit SGN.Q as ALU 406C (CQ). This arrangement results in the value of ALU 406C (CQ) being incremented by the output value of table 402 on TRUE SGN.Q, but decremented by twice the output value of table 402 on FALSE SGN.Q.

The output of sine table 404 is connected to the inputs of ALUs 406F (SD), 406G (SL), 406I (S24), and 406J (S13), bit shifter 412, and multiplexer 414. A second input (input B) of multiplexer 414 is the output of sine table 404 multiplied by 2 through shifting by 1 bit with bit shifter 412. The multiplexed position-data output by multiplexer 414 is connected to ALU 406H (SQ). Multiplexer 414 is steered by the same control bit SGN.Q as ALU 406H (SQ). This arrangement results in the value of ALU 406H (SQ) being incremented by the output value of table 404 on TRUE SGN.Q, but decremented by twice the output value of table 404 on FALSE SGN.Q.

In addition to processing position-data words 300 and measurement channel phase words 209M, block data generator 210 also synthesizes three 320 clock length (1024 microseconds) digital sequences *I*, *J*, and *K*. The digital sequences *I*, *J*, and *K*, are repeated once per group 600 of 320 clocks at 3.2 microseconds, or 1024 microseconds.

Sequence I is a constant logical 1, and is connected to the inputs of ALUs 406P (I24) and 406Q (I13). Sequence I is one bit wide. Therefore, ALUs 406P (I24) and 406Q (I13) could be simple counters in an alternative embodiment. Sequence I sums to 2^8 under Operation D, and zero under operations L and Q. Operations D, L and Q are summarized above in Table II.

Sequence J linearly progresses from -159.5 to $+159.5$ by $+1$ per step. Sequence J comprises 9 bits from up-counter 422 concatenated with 1 least significant bit of a constant logical 1. Sequence J is 10 bits wide and connects to the inputs of ALUs 406R (J24) and 406S (J13). Sequence J sums to 2^{14} under Operation L, and zero under operations D and Q.

Sequence K is quadratic, and numerically equals $J^2 - 9{,}045.25$ at each step. Sequence K is 16 bits wide, and is connected to the inputs of ALUs 406T (K24) and 406U (K13). The value 9,045.25 equals $(64 \cdot 127 \cdot 129 - 96 \cdot 191 \cdot 193 + 160 \cdot 321 \cdot 319)/(12 \cdot 128)$, which ensures that K sums to zero when blocks 1, 2, 4, 5, 6, 7, 9, and 10, are summed. Sequence K is synthesized by loading 16,395 into accumulator 426 at the beginning of a new group 600 of position-data words 300, and accumulating $(2 \cdot J - 1)$ at each step, which is twice the value of counter 422 alone, that is, without the 1 concatenated least significant bit. J is multiplied by 2 through shifting J by 1 bit with bit shifter 424. Sequence K sums to 2^{21} under Operation Q, and zero under operations D and L.

The terminal count output of modulo-5 counter 506 generates a group clock signal every 1024 microseconds, which indicates the end of a group 600 of position-data words 300. The group clock signal from modulo-5 counter 506 is coupled to 21 registers 421. Each of the 21 registers 421 is also coupled to the output of one of the 21 ALUs 406. When the group clock signal is received by registers 421, all 21 outputs of the ALUs 406 are latched to the 21 registers 421. ALUs 406 are then reset to zero, and up-counter 422 and accumulator 426 are loaded with values 160 and 16,395, respectively. After being reset, ALUs 406 are once again receptive to new entries.

C. Generation of Non-Linearity Parameters (Magnitude V_{NL} and Phase θ_{NL})

The 21 values latched by registers 421 from ALUs 406 are I_{24}, J_{24}, K_{24}, C_{24}, S_{24}, P_{24}, I_{13}, J_{13}, K_{13}, C_{13}, S_{13}, P_{13}, P_D, P_L, P_Q, C_D, C_L, C_Q, S_D, S_L, and S_Q, which are output by ALUs 406P, 406R, 406T, 406D, 406I, 406N, 406Q, 406S, 406U, 406E, 406J, 406O, 406K, 406L, 406M, 406A, 406B, 406C, 406F, 406G, and 406H, respectively. The 21 values are digitally processed by parameter generator 212 to produce the magnitude V_{NL} and phase θ_{NL} of any non-linearity present in the signal using the mathematical procedure shown in the following Equations IX through XIX.

Six quantities, P_{24}', C_{24}', S_{24}', P_{13}', C_{13}', and S_{13}', are generated as follows:

Equation IX

$$P_{24}' = P_{24} - \frac{I_{24} \cdot P_D}{2^8} - \frac{J_{24} \cdot P_L}{2^{14}} - \frac{K_{24} \cdot P_Q}{2^{21}}$$

Equation X

$$C_{24}' = C_{24} - \frac{I_{24} \cdot C_D}{2^8} - \frac{J_{24} \cdot C_L}{2^{14}} - \frac{K_{24} \cdot C_Q}{2^{21}}$$

Equation XI

$$S_{24}' = S_{24} - \frac{I_{24} \cdot S_D}{2^8} - \frac{J_{24} \cdot S_L}{2^{14}} - \frac{K_{24} \cdot S_Q}{2^{21}}$$

Equation XII

$$P_{13}' = P_{13} - \frac{I_{13} \cdot P_D}{2^8} - \frac{J_{13} \cdot P_L}{2^{14}} - \frac{K_{13} \cdot P_Q}{2^{21}}$$

Equation XIII

$$C_{13}' = C_{13} - \frac{I_{13} \cdot C_D}{2^8} - \frac{J_{13} \cdot C_L}{2^{14}} - \frac{K_{13} \cdot C_Q}{2^{21}}$$

Equation XIV

$$S_{13}' = S_{13} - \frac{I_{13} \cdot S_D}{2^8} - \frac{J_{13} \cdot S_L}{2^{14}} - \frac{K_{13} \cdot S_Q}{2^{21}}$$

From the six quantities calculated in Equations IX through XIV, non-linearity parameters V_{NL} and θ_{NL} are generated as shown in the following Equations XV through XIX:

Equation XV

$$\det = -C_{24}' \cdot S_{13}' + C_{13}' \cdot S_{24}'$$

Equation XVI

$$A_{NL} = -S_{13}' P_{24}' + S_{24}' P_{13}'$$

Equation XVII

$$B_{NL} = C_{13}' P_{24}' - C_{24}' P_{13}'$$

Equation XVIII

Non-linearity magnitude $V_{NL} = (A_{NL}^2 + B_{NL}^2)^{1/2}/\det.$

Equation XIX

Non-linearity phase $\theta_{NL} = \text{arctangent}(B_{NL} / A_{NL})/2\pi$

Both the non-linearity magnitude V_{NL} and phase θ_{NL} are expressed in UI, where one UI represents $\lambda/4$. The peak position-deviation from ideal is V_{NL} and the location of the periodic pattern is given by the phase θ_{NL}.

In one embodiment, only fixed-point computations are performed by compensation system 200, and division operations are limited to powers of 2. In one form of the invention, the process performed by compensation system 200 is implemented with either hardware or firmware, or a combination of hardware and firmware.

D. Non-Linearity Compensation Using Generated Parameters

The non-linearity pattern that is a subject of embodiments of the present invention is a result of single-side-band (SSB) modulation and is not a single sinusoid. Specifically, $\Delta\phi(\phi)$, the phase deviation from ideal as a function of non-linearity phase ϕ, is given in UI by the following Equation XX:

Equation XX

$$\Delta\phi(\phi)=\left(\frac{1}{2\pi}\right)\arctan\left(\frac{r\cdot\cos[2\pi(\phi-\theta)]}{1-r\cdot\sin[2\pi(\phi-\theta)]}\right)$$

where:

r is the ratio of the perturbing signal magnitude to the ideal signal magnitude; and

Angle θ is a phase difference between the non-linearity periodicity $\Delta\phi(\phi)$ and the non-linearity phase progression ϕ. The block regression process measures the phase offset as θ_{NL}.

For small deviations, $\Delta(\phi)$ is well approximated by the following Equations XXI through XXIV:

Equation XXI

$$\Delta\phi(\phi) \approx \left(\frac{1}{2\pi}\right)\left(\frac{r\cdot\cos[2\pi(\phi-\theta)]}{1-r\cdot\sin[2\pi(\phi-\theta)]}\right)$$

Equation XXII

$$\Delta\phi(\phi) = \frac{\left(\frac{r}{2\pi}\right)\cdot\cos[2\pi(\phi-\theta)]}{1-2\pi\cdot\left(\frac{r}{2\pi}\right)\sin[2\pi(\phi-\theta)]}$$

Equation XXIII

$$\Delta\phi(\phi) \approx V_{NL}\cdot\cos 2\pi(\phi-\theta_{NL})\cdot\left[1+2\pi\cdot V_{NL}\sin 2\pi(\phi-\theta_{NL})\right]$$

Equation XXIV

$$\Delta\phi(\phi) \approx V_{NL}\cdot\cos 2\pi(\phi-\theta_{NL})+\pi\cdot V_{NL}{}^{2}\sin 4\pi(\phi-\theta_{NL})$$

The quantity $r/2\pi$ in Equation XXII is measured as V_{NL}, the magnitude, in Equations XXIII and XXIV, and θ in Equation XXII appears as deviation phase, θ_{NL}, in Equations XXIII and XXIV. Both quantities V_{NL} and θ_{NL} are determined in one embodiment using the block regression method described above. The values of V_{NL} and θ_{NL} are estimated accurately using only the cosine term. The sine term, being orthogonal at exactly twice the frequency, need not be involved in the block regression estimation, but plays a part in determining the actual compensation value.

Even though the non-linearity parameters (magnitude-V_{NL}, and phase-θ_{NL}) are derived from measured data that arrive at a rate of one every 3.2 microseconds, they may be used to compensate position data at any rate for the next 1024 microseconds.

The following Equation XXV can be described as the phase deviation $\Delta\phi(j)$ at any time step j, as a function of the ideal non-linear phase $\phi(j)$:

Equation XXV

$$\Delta\phi(j) \approx V_{NL} \cdot \cos 2\pi\phi(j) + \pi \cdot V_{NL}^{\,2} \sin 4\pi\phi(j)$$

where V_{NL} is a measured value.

Equation XXV shows the expected non-linearity phase deviation $\Delta\phi(j)$ given the ideal phase $\phi(j)$. The value θ_{NL} is also a measured value. It is subtracted from the non-linearity phase progression ($\psi_{NL}(j)$) to generate an offset-removed non-linearity phase progression $\phi_{NL}(j)$. In practice, the ideal $\phi(j)$ is not available. But $\phi_{NL}(j)$ itself is subject to non-linearity perturbation and is different from the ideal $\phi(j)$ by an unknown amount $\Delta\phi(j)$ as shown in the following Equation XXVI:

Equation XXVI

$$\phi_{NL}(j) = \phi(j) + \Delta\phi(j)$$

The phase deviation $\Delta\phi_{NL}(j)$ based on the corrupted argument $\phi_{NL}(j)$ is:

Equation XXVII

$$\Delta\phi_{NL}(j) \approx V_{NL} \cdot \cos 2\pi\phi_{NL}(j) + \pi \cdot V_{NL}^{\,2} \sin 4\pi\phi_{NL}(j)$$

This quantity in Equation XXVII, however, is not exactly the compensation needed as it is based on a corrupted argument.

Assuming changes are piecewise linear near the region between $\phi_{NL}(j)$ and $\phi(j)$, these two are also related as shown in the following Equation XXVIII:

Equation XXVIII

$$\Delta\phi(j) = \Delta\phi_{NL}(j) - \Delta'\phi_{NL} \cdot \Delta\phi(j)$$

The desired deviation $\Delta\phi(j)$ can be solved as:

Equation XXIX

$$\Delta\phi(j) = \frac{\Delta\phi_{NL}(j)}{1 + \Delta'\phi_{NL}} = \frac{V_{NL} \cdot \cos[2\pi\phi_{NL}(j)] + \pi \cdot V_{NL}^{\,2} \sin[4\pi\phi_{NL}(j)]}{1 - 2\pi V_{NL} \sin[2\pi\phi_{NL}(j)] + 4\pi^2 V_{NL}^{\,2} \cos[4\pi\phi_{NL}(j)]}$$

For small values of V_{NL} (*e.g.*, $V_{NL} << 1/2\pi$), making use of $1/(1-\delta) \approx 1+\delta$, and dropping all terms involving V_{NL}^3 or higher, $\Delta\phi(j)$ may be simplified as shown in the following Equations XXX and XXXI:

<u>Equation XXX</u>

$$\Delta\phi(j) \approx \left(V_{NL}\cos[2\pi\phi_{NL}(j)] + \pi V_{NL}^{\ 2}\sin[4\pi\phi_{NL}(j)]\right)\cdot\left(1 + 2\pi V_{NL}\sin[2\pi\phi_{NL}(j)]\right)$$

<u>Equation XXXI</u>

$$\Delta\phi(\phi_{NL}) = V_{NL}\cdot\cos 2\pi\phi_{NL} + 2\pi\cdot V_{NL}^{\ 2}\cdot\sin 4\pi\phi_{NL}$$

Therefore, the desired phase deviation $\Delta\phi(j)$ may be constructed from the corrupted phase $\phi_{NL}(j)$ quite accurately up to the second harmonics by Equation XXXI.

In one embodiment, $\Delta\phi$ is computed into a single read/write memory table 206A (shown in Figure 7), having 256 entries spanning one period of the ϕ_{NL}, and addressable by the 8 most significant fractional bits, *NLF1-NLF8* of ϕ_{NL}. Each entry in table 206A is 8 bits wide in accordance with equation XXXI.

Figure 7 is an electrical block diagram illustrating in greater detail the compensation processing block 206 shown in Figure 2. Compensation processing block 206 includes RAM table generator 702, RAM table 206A, and ALU 206B. In one form of the invention, RAM table 206A includes 256 values for $\Delta\phi$, calculated from Equation XXXI, using fractional ϕ values that span one period. The various values for $\Delta\phi$ in RAM table 206A are represented by $\Delta\phi(\phi_{NL})$, where the relationship is as in Equation XXXI. RAM table 206A is generated by RAM table generator 702 based on non-linearity parameter, V_{NL}, provided by parameter generator 212. The other non-linearity parameter θ_{NL}, also provided by parameter generator 212, is subtracted from current observed non-linearity data words, $\psi_{NL}(j)$, by ALU 217 to generate the non-linear phase progression $\phi_{NL}(j)$. RAM table generator 702 updates the $\Delta\phi(j)$ values stored in RAM table 206A each time a new value for V_{NL} is received. As described above, V_{NL} and θ_{NL} are calculated every 1024 microseconds (i.e., every 320

position-data words 300 and the corresponding measurement channel phase progression words 209M).

At any given time-index, j, the (corrupted) phase $\phi_{NL}(j)$ will address the table 206A. RAM table 206A is addressable by the 8 most significant fractional bits *NLF1-NLF8* of the non-linear phase progression $\phi_{NL}(j)$. The $\Delta\phi$ values as addressed by the non-linearity phase $\phi_{NL}(j)$ at a 3.2 microsecond rate are output by RAM table 206A to ALU 206B. The value output by table 206A is precisely the compensation needed at the moment. ALU 206B subtracts the value output by table 206A from the current position-data word to get the compensated data word regardless of the rate of position-data words being compensated. ALU 206B subtracts the $\Delta\phi$ values received from RAM table 206A from received position data, including, in one embodiment, measured or observed position data and extrapolated position data.

The non-linearity progression $\phi_{NL}(j)$, which is derived from observed position data by subtracting an offset θ_{NL}, determines the non-linearity present at any time, irrespective of the data rate of the uncompensated position data. The mathematical representation of the compensation process for extrapolated position data is shown in the following Equation XXXII:

Equation XXXII

$$p_{Ideal}(i) = p(i) - V_{NL} \cdot \cos 2\pi\phi_{NL}(j) - 2\pi \cdot V_{NL}^{2} \cdot \sin 4\pi\phi_{NL}(j)$$

where

i stands for any data rate; and

j stands for the observed data rate.

E. Update Suspension of Non-Linearity Parameters

In one embodiment, when the measurement channel frequency approaches half the reference channel frequency to within ±7.8125 kHz, or within multiples of 312.500 kHz ± 7.8125 kHz, the updating of the quasi-static non-linearity parameters θ_{NL} and V_{NL} is suspended. When one of these velocity conditions is present, block-data generator 210 (shown in Figure 2) sends an

update inhibit signal 216 to parameter generator 212, indicating that updating of the non-linearity parameters is to be suspended. In one form of the invention, the test for the absence of the above frequency conditions is: (1) A group of 320 non-linearity phase data words 219 must have at least 8 "quadrant" transitions within 1024 microseconds; and (2) the group of 320 non-linearity phase data words 219 must not dwell on any one quadrant for more than 128 microseconds.

A "quadrant" is defined by the most significant two bits (*NLF1* and *NLF2*) of the fractional part of non-linearity phase data words as shown in the following Table IV:

Table IV

(*NLF1*, *NLF2*)	Name
0, 0	1st quadrant
0, 1	2nd quadrant
1, 0	3rd quadrant
1, 1	4th quadrant

The non-linearity parameters θ_{NL} and V_{NL} are updated when one or both of the above two tests are satisfied. If neither test is satisfied, updating of the non-linearity parameters is suspended. The non-linearity parameter values calculated prior to suspension are held and used to compensate new position data in an uninterrupted manner. Updating of the non-linearity parameters resumes only when either or both of the above two tests are satisfied.

In some metrology systems, when the difference between twice the measurement channel frequency and the reference channel frequency is high, a tracking filter eliminates the perturbing effect of the leakage signal, and there is no significant non-linearity present in the data. One embodiment of the present invention bypasses the compensation process during such high frequency difference conditions.

Embodiments of the present invention are applicable primarily to heterodyne interferometers. In one embodiment, the interferometer itself is not modified in any way. Rather, the resulting measurement data output by the interferometer is processed, and two best-fit quasi-static non-linearity parameters

are produced to compensate data in the near future. Continuing the process indefinitely, all data after an initial latency period are compensated. Embodiments of the invention involve only digital numerical computation by hardware, firmware, or a combination. In one form of the invention, no optical or analog electrical circuitry is used. No Fourier transformation or other spectral analysis methods are employed in one embodiment.

Although specific embodiments have been illustrated and described herein for purposes of description of the preferred embodiment, it will be appreciated by those of ordinary skill in the art that a wide variety of alternate and/or equivalent implementations may be substituted for the specific embodiments shown and described without departing from the scope of the present invention. Those with skill in the chemical, mechanical, electro-mechanical, electrical, and computer arts will readily appreciate that the present invention may be implemented in a very wide variety of embodiments. This application is intended to cover any adaptations or variations of the preferred embodiments discussed herein. Therefore, it is manifestly intended that this invention be limited only by the claims and the equivalents thereof.